USE OF FUZZYCONES FOR SUN-ONLY ATTITUDE DETERMINATION: THEMIS BECOMES ARTEMIS

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ABSTRACT

In order for two THEMIS probes to successfully transition to ARTEMIS it will be necessary to determine attitudes with moderate accuracy using Sun sensor data only. To accomplish this requirement, an implementation of the Fuzzycones maximum likelihood algorithm was developed. The effect of different measurement uncertainty models on Fuzzycones attitude accuracy was investigated and a bin-transition technique was introduced to improve attitude accuracy using data with uniform error distributions. The algorithm was tested with THEMIS data and in simulations. The analysis results show that the attitude requirements can be met using Fuzzycones and data containing two bin-transitions.

INTRODUCTION

The Fuzzycones method is a maximum likelihood algorithm for combining angle measurements into vectors. It has been demonstrated for combining the angles determined by Coarse Sun Sensor eyes into a Sun direction vector [1] and for combining measurements of the angles from a spacecraft spin-axis into the direction of that axis [2].

The Time History of Events and Macroscale Interactions during Substorms (THEMIS) mission consists of five probes that were launched on February 17, 2007 by NASA. After two years of successful operations by the University of California, Berkeley's Space Science Laboratory, it has been proposed to repurpose two of the probes to a new mission: "Acceleration, Reconnection Turbulence, and Electrodynamics of Moon's Interaction with the Sun" (ARTEMIS).

In the Artemis mission, Themis probes B and C, will be moved from their current elliptical orbits (Probe B: 1.2 x 31.7 R_E, Probe C: 1.3 x 19.6 R_E) into equatorial orbits around the Moon.

Artemis will [3]

> "… make the first systematic, two-point observations of distant magnetotail phenomena with comprehensive instrumentation. ARTEMIS will resolve outstanding questions regarding particle acceleration, reconnection and turbulence in the magnetotail and the solar wind, and study the formation and dynamics of the lunar wake from 1500km–30RL."

The Themis probes are spin-stabilized and use a Three-axis Magnetometer (TAM) and a Digital Sun sensor (DSS). During the mission, data from these instruments was used by a Spinning-Spacecraft Kalman Filter [4] (SpinKF) to determine attitude. SpinKF uses the TAM data only near spacecraft perigee, when the magnetic field is significant and the Earth magnetic field models are reliable. The spin-axis attitudes have been found to be very stable throughout the mission and are nominally about 8 degrees from the ecliptic pole*.

During the first few days after launch data was very sparse and no reliable TAM data was available. A prototype Fuzzycones algorithm was implemented and used to verify the expected attitude using only DSS measurements.

When the ARTEMIS probes leave the vicinity of the Earth, the TAMs will no longer be useable for attitude determination. Attitudes will have to be determined using the DSS alone. This limitation presents only minor problems during the science periods. The spin-axis attitude is stable enough that the relatively large Sun position changes over the many days of science observations can be used to compute attitudes that are accurate enough for use.

When the spacecraft is maneuvered the situation is different. Each maneuver changes the attitude significantly. Maneuvers may need to be scheduled with short intervals between them. Attitude errors during any maneuver decrease the efficiency of use of the limited remaining fuel.

This paper describes the use of Fuzzycones with data from a DSS only to determine spin-axis attitudes. In particular, the accuracy of Artemis attitudes is described as a function of the time interval used to obtain the attitudes. The ability to obtain accurate attitudes using a relatively short time span of DSS data was a requirement of converting THEMIS to ARTEMIS.

FUZZYCONES ALGORITHM

Fuzzycones uses a set of measured angles from directions and the corresponding directions in a reference frame to compute the direction of a solution vector in the reference frame. For spin-axis attitude determination, it uses measurements of angles from the spin axis to directions that are known in an inertial frame to compute the direction of the spin axis in the reference frame. For Sun-only attitude determination it uses the known spin-axis in the body frame, measured angles of the Sun from this axis, and spacecraft-Sun vectors in an inertial frame (computed from ephemerides) to obtain the spin-axis vector in the inertial frame.

For many years the cones method [5],[6], [7] has been used to compute solution vectors from pairs of measurements and reference vectors. Fuzzycones is conceptually derived from cones but the mathematics is quite different.

In this paper reference vectors are designated $\hat{V}_i$, corresponding measurement angles, ρ_i, and the uncertainties of these measurements η_i. The measurements represent angles between the reference vector and a solution vector.

Any point on a unit sphere can be represented in terms of its azimuth, ϕ, and elevation, θ, in some coordinate system. In particular, we represent the position of each reference vector by Φ_i and Θ_i.

A fuzzycone, F_i, is defined as the probability that the value measured at a point represents the solution vector.

$$F_i(\phi,\theta) = \mathbf{P}_i(\Delta\Gamma_i, \eta_i) \tag{1}$$

where $\Delta\Gamma_i$ is the measurement difference:

$$\Delta\Gamma_i = \Gamma_i - \rho_i \tag{2}$$

* Coordinates in this paper will be described in spacecraft-centered ecliptic coordinates defined with the X-Y plane parallel to the ecliptic plane and the Z-axis parallel to the ecliptic pole.

and Γ_i is the angle from the point, (ϕ,θ), to the reference vector (Φ_i,Θ_i).

Γ_i is given by:

$$\Gamma_i = \cos^{-1}\left(\cos\Phi_i \sin\phi + \sin\Phi_i \sin\phi \cos(\Theta_i - \theta)\right) \tag{3}$$

The single fuzzycone is a function of this difference between the angle from the reference vector and the measured angle.

The Fuzzycones function is the product of the fuzzycones representing each of the measurements:

$$\mathscr{F} = \prod_i F_i \tag{4}$$

The Fuzzycones function represents the probability that the solution vector is at a point on the unit sphere and its maximum represents the most probable solution.

MEASUREMENT UNCERTAINTIES

In order to determine the maximum of $\mathscr{F}$, the probability as a function of measurement difference and measurement uncertainty: $\mathbf{P}_i(\Delta\Gamma_i,\eta_i)$ must be defined.

The most common uncertainty models* are Gaussian (Normal) and uniform distributions. Gaussian models are by far the most frequently used, not necessarily because they represent the uncertainties better, but often because they are more amenable to analytical representation.

The Gaussian probability function is given by:

$$\mathbf{G}_i(x,\sigma_i) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(x^2/2\sigma^2\right)} \tag{5}$$

where x is a variable with uncertainty of σ. The term $\frac{1}{\sigma\sqrt{2\pi}}$ is a normalization factor which is not needed in the Fuzzycones algorithm and for clarity, will not be used in the following equations.

For Fuzzycones, x is replaced by $\Delta\Gamma_i$, and σ by η_i.

Gaussian functions are defined for x having a range of $\pm\infty$, but the calculated and measured angles (Γ_i and ρ_i) are restricted to the range of 0 to 2π. A rigorous expression for the Gaussian probability function used in Fuzzycones is:

$$\mathbf{G}_\mathbf{i} = \sum_{j=0}^{\infty} e^{-\frac{(\Gamma_i - j(2\pi) - \rho_i)^2}{2\eta_i^2}} + \sum_{j=0}^{\infty} e^{-\frac{(\Gamma_i + j(2\pi) + \rho_i)^2}{2\eta_i^2}} \tag{6}$$

In practice values of j greater than zero are always negligible so:

$$\mathbf{G}_\mathbf{i} = e^{-\frac{(\Gamma_i - \rho_i)^2}{2\eta_i^2}} + e^{-\frac{(\Gamma_i + \rho_i)^2}{2\eta_i^2}} \tag{7}$$

* The term "uncertainty models" rather than "noise models" is used in this paper because measurement uncertainty can, and often, does arise from other sources (for example model inaccuracies) than true, random noise.

When measured angles are significantly larger than their uncertainties ($\rho_i >> \eta_i$) then the second term is also negligible.

Uniform distributions are defined as having a constant value within a range of variable values and zero outside the range. The most common example of uniform distributions are digital sensors. In these all measurements with values over a defined range result in identical reported measurements. The reported measurements are the value of the "bin" in which the true value falls and are within ½ of a range corresponding to the size of the least significant bit from the measured value (the bin width). If a detector's least significant bit corresponds to an angle of δ then an observed measurement of ρ_i means that the probability that the true value resulting in the measurement is equal in the range of $\rho_i - \delta/2$ to $\rho_i + \delta/2$.

FUZZYCONES WITH MEASUREMENT UNCERTAINTIES

A single fuzzycone modeled with Gaussian measurement uncertainties is shown in Fig. 1. The probability is on an arbitrary scale.

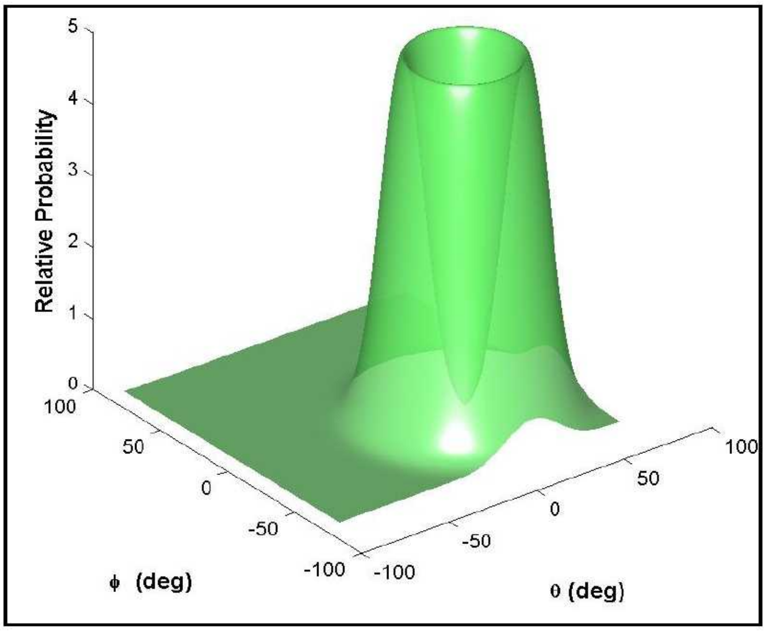

Fig. 1. A Single Fuzzycone with Gaussian Uncertainties

Fig. 2 shows a schematic representation of the intersection of three fuzzycones representing measured angles from three distinct axes. In this figure the three individual fuzzycones are shown with heights adjusted to improve visibility. The central "peak" represents the product of the fuzzycones. To the right of the fuzzycones representation is a contour plot representing the product function only.

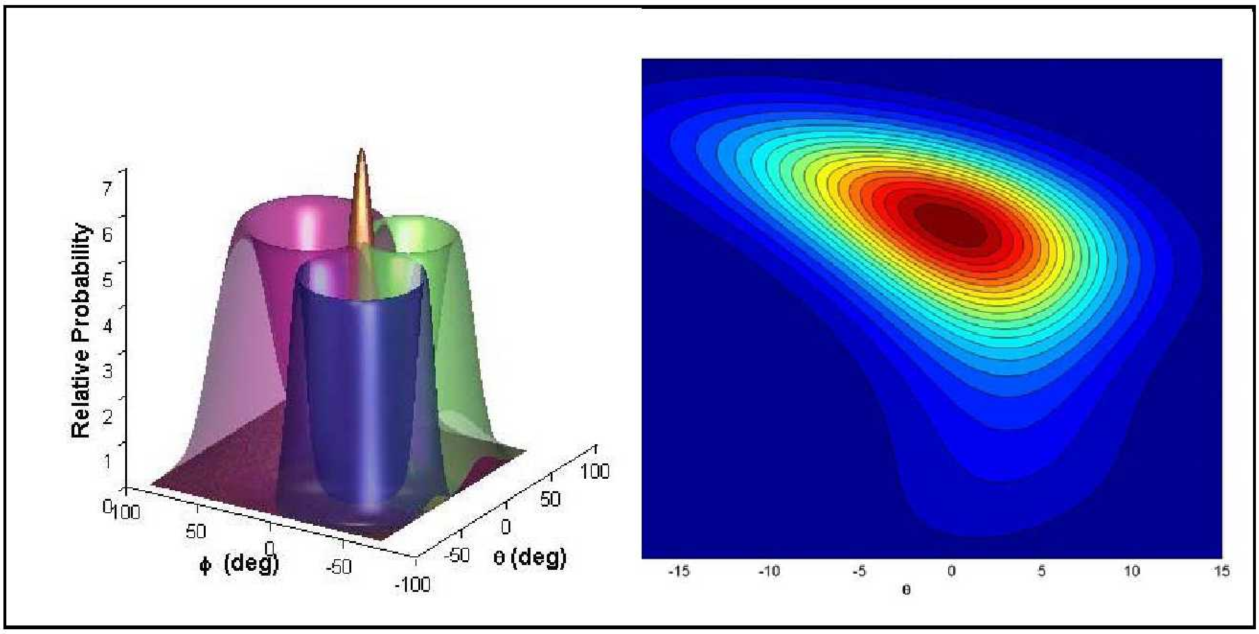

Fig. 2. The Product of Three Fuzzycones Representing Measurements From Three Separate Axes

It is much harder to represent fuzzycones with uniform distribution. Since each fuzzycone represents constant probability over a specified range the product of two fuzzycones represents a quadrilateral region on the surface of the unit sphere, in which the probability is constant, and outside of which the probability is zero.

Often, uniform distributions are replaced by equivalent Gaussian distributions for computational simplicity. A uniform distribution of width δ has the same standard deviation as a Gaussian distribution with $\sigma = \delta / \sqrt{12}$.

SUN-ONLY ATTITUDES AND UNIFORM UNCERTAINTY

The DSS on the THEMIS probes are digitized at 0.125 degrees. The DSS measurement uncertainty is dominated by this uniform distribution.

For Sun-only attitudes on spinning spacecraft the reference directions are the vectors from the spacecraft to the Sun in inertial coordinates and the measurements are the angles from the spacecraft spin axis to the Sun. The assumption is made that the spin axis is invariant in inertial space over the data time span.

When attempting to determine Sun-only attitudes over a relatively short period (a few days) the reference vectors and angles change by at most a few degrees. The quadrilateral region representing a non-zero probability of the spin-axis is defined by only the extreme values. The product of the first and last uniform distribution fuzzycones represents the smallest subset of the regions defined by the products of any other pair.

The uniform probability region for uniform distribution can be easily examined in two extreme cases: when the spin axis is in the ecliptic plane and when the spin axis is at the ecliptic pole.

When the spin axis is in the ecliptic plane the measured angle changes with time at exactly the same rate as the Sun position. With perfect measurements, cones from the Sun directions to the spin axis will be tangent to each other at the spin-axis location. This situation is illustrated in Fig. 3.

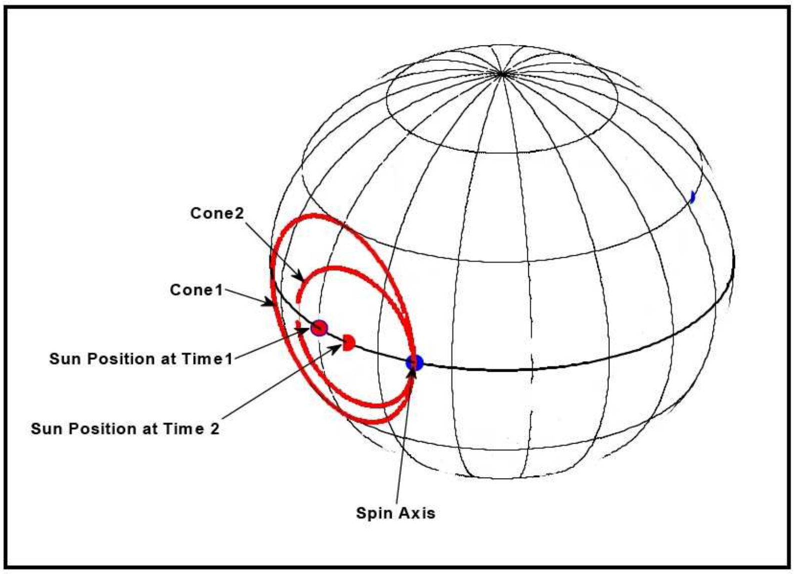

Fig. 3. Intersection of Two Sun Cones with Spin-axis in the Ecliptic Plane

With uniform measurement uncertainty the cones are replaced by annular regions on the unit sphere, the width of which is the measurement uncertainty. The intersection region is illustrated in Fig. 4.

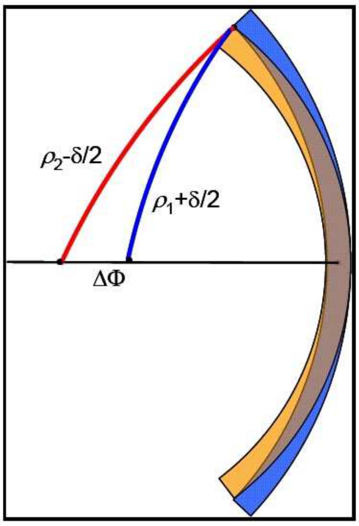

Fig. 4. Intersection of Two Cones with Uniform Uncertainty and Spin-axis in the Ecliptic Plane

In Fig. 4 the measured Sun-axis to Sun angles are ρ_1 and ρ_2 , the spacecraft to Sun position changes by $\Delta\Phi$ (which in this case is the same as $\Delta\rho$ and which represents the change in Sun position over the measurement span), and the measurement uncertainty is δ. The short diagonal in the intersection quadrilateral is close to δ. The long diagonal can be calculated numerically and for δ = 0.125 degrees. Cases were taken for a series of values of ρ_1 and $\Delta\rho$ and the long diagonal (maximum error) from the intersection region determined numerically. In Fig. 5 these maximum errors are plotted against ρ_1 for a series of values of $\Delta\rho$ and in Fig. 6 maximum errors are plotted against $\Delta\rho$ for a series of values of ρ_1.

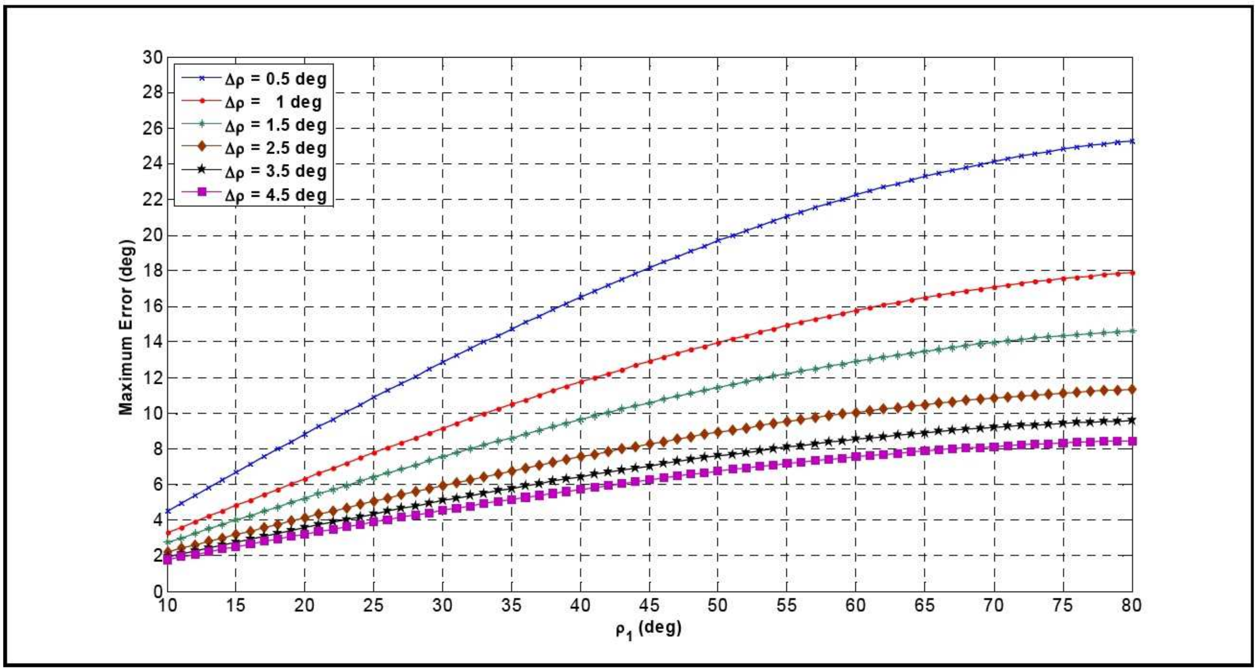

Fig. 5. Spin-axis in Ecliptic Case: Maximum Spin-axis Determination Error as a Function of Angle from Sun for Several Changes in Sun Position

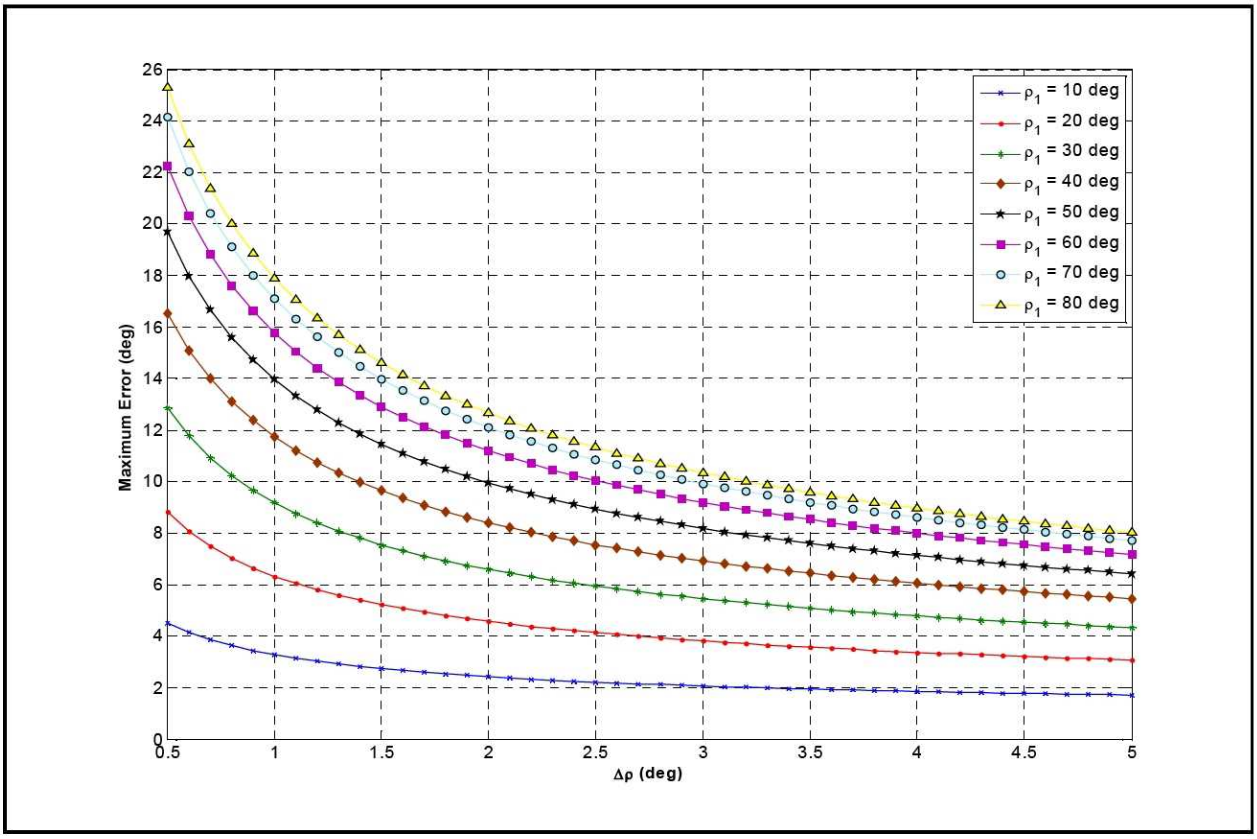

Fig. 6. Spin-axis in Ecliptic Case: Maximum Spin-axis Determination Error as a Function of Changes in Sun Position for Several Initial Sun Angles

As can be seen from these figures the maximum errors are quite large, except for large Sun position changes (long time spans) or if the Sun direction is near the spin-axis.

When the spin axis is at the ecliptic pole, the measured Sun angle must be within $\pm\delta/2$ of 90 degrees. Over time, the Sun position changes but the measured angle does not. This situation is illustrated in Fig. 7.

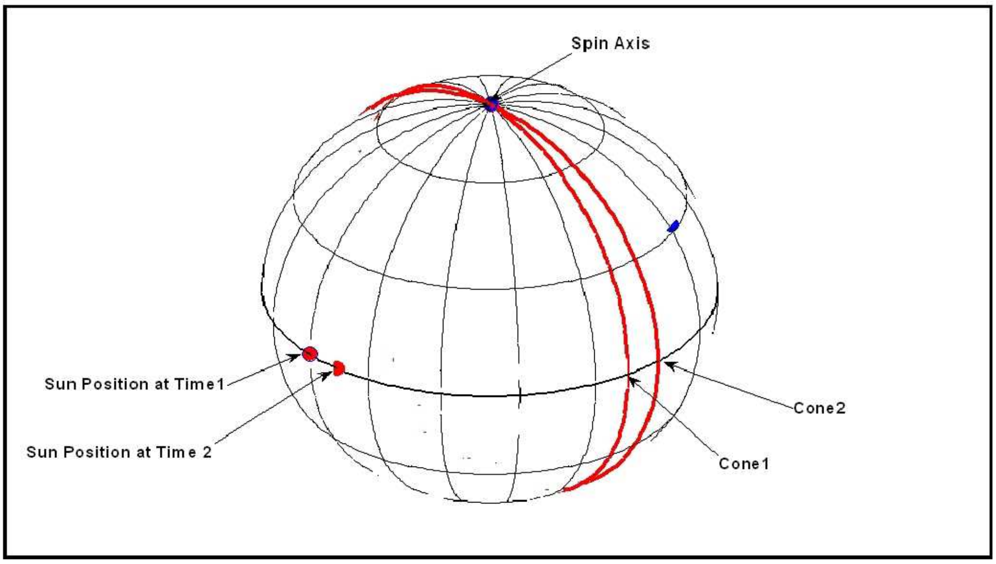

Fig. 7. Intersection of Two Sun Cones with Spin-axis at the Ecliptic Pole

With uniform measurement uncertainty one of the two intersection regions is illustrated in Fig. 8.

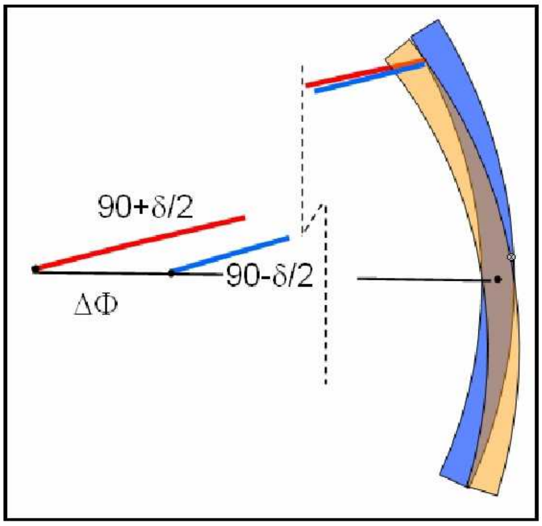

Fig. 8. Intersection of Two Cones with Uniform Uncertainty and Spin-axis at the Ecliptic Pole (note the discontinuity in the representation)

With the spin-axis at the pole the minimum and maximum errors (the diagonals of the intersection region) can be estimated by:

$$Error_{\min} \approx \frac{\delta}{\cos\left(\Delta\Phi/2\right)} \tag{8}$$

$$Error_{\max} \approx \frac{\delta}{\sin\left(\Delta\Phi/2\right)} \tag{9}$$

From Eq. (9) for a DSS with δ of 0.125 degrees, a value of $\Delta\Phi$ of 2 degrees results in a maximum error of about 7.2 degrees. Although this error is large, it is generally smaller than the errors in the spin-axis on the ecliptic case.

There are equivalent regions of intersection at each of the ecliptic poles.

BIN TRANSITIONS

The THEMIS/ARTEMIS spin-axis is maintained at approximately 82 degrees from the ecliptic normal so the errors expected due to the uniform distribution of DSS measurement data will be closer to those in the spin-axis at the ecliptic pole case than those in the spin-axis in the ecliptic case. Nevertheless, the expected errors will be larger than required.

To reduce the errors further a bin transition technique was evaluated and eventually used. Because the THEMIS/ARTEMIS spin-axes are not exactly at the ecliptic pole, the measured Sun angle changes with time. Although measurements of all Sun angles in any 0.125 degree bin are identical, the measured value changes at a bin transition—a region where the digitization changes and the Sun angle changes from one bin to another. Such a case is shown in Fig. 9.

This figure shows the measured Sun angle as a function of time. In the region outlined in red the angle changes from the 82.02 degree bin to an 81.895 degree bin. Although the time between bin transitions is several days, the time of the bin transition itself is measureable to better than a few minutes (See inset in Fig. 9). During this short period, the Sun angle is very close to half-way between the angles representing the two bins: 81.9575 degrees.

From Fig. 9 it is clear that the bin transition does not occur instantaneously. The pattern of moving from one bin to the other and back can be modeled as having small Gaussian uncertainties simultaneous with the known, larger, uniform uncertainties. The measured time range and measured distribution of Sun angles over the time allows an estimate of standard deviation of the Gaussian component of the error to be made. For THEMIS data, the standard deviation of the Gaussian portion of the error was found to be about 0.001 degree.

Several ways of using this bin transition information have been tried. Attempts were made to average the times, fit the times and bin values linearly, and simply take the bin mean at the time halfway between the first and last instance of bin change during the period containing the bin transition. All of these worked equally well.

Because Fuzzycones is particularly good at using multiple measurements, and in order to make the method general, the final algorithm defined two types of Sun angle data. Normal Sun angle data included the direct measurements and was represented as Gaussian with a standard deviation equal to the bin size divided by the square root of 12. Bin transition Sun (BTS) angle data was found from the mean times and mean angles of each bin transition and were assumed to have a standard deviation of 0.001 degree.

If a single bin transition was used, the BTS angles were given time tags of each normal Sun angle measurement from the first to the last bin change in each bin transition period.

If two or more bin transitions were found then their mean angle and mean time were used to construct a time/angle straight line for each pair. At each input Sun angle time, the angles along these lines were used as the BTS angles at the times.

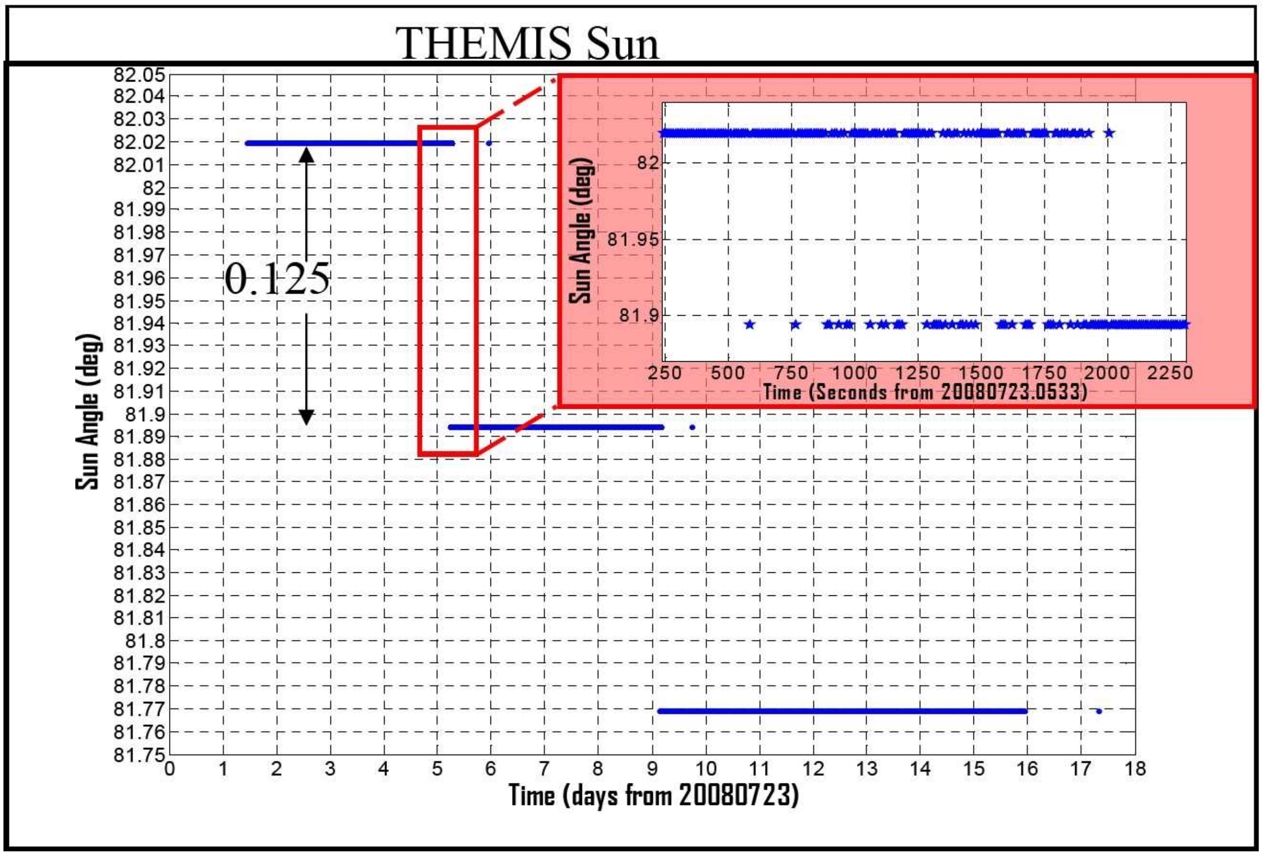

Fig. 9. Sun Sensor Bin Transitions in THEMIS Data

Using THEMIS data, the accuracy of Fuzzycone attitudes was estimated for several cases by comparing with nominal SpinKF attitudes.

If BTS data is not used, the Fuzzycones attitude errors depend on the length of the span used and are as much as about 3 degrees. If data from two bins are used, this value drops to about 1 degree. If a pair of bin transitions were used, the BTS data results in Fuzzycones solutions with errors of about 0.3 deg. These error estimates are uncertain because the SpinKF uncertainty is on the same order of magnitude as the differences between SpinKF and two-bin-transition fuzzycones results*. These results are summarized in Table 1.

Table 1. THEMIS Attitude Differences Between Fuzzycones Solutions and the Nominal SpinKF Solution

Start Date (mm/dd in 2008) / Case	07/17	07/20	07/24	07/28	08/01	08/07
One Bin	3.33	2.99	2.37	1.85	1.12	0.16
Two Bins	1.07	1.04	0.25	0.21	0.15	n/a
Two Bin Transitions	0.41	0.47	0.19	0.20	0.20	n/a

* The error in SpinKF attitudes is not a reflection on the accuracy of the method. The dominant sensor input to SpinKF for THEMIS is magnetometer data. Magnetometer data is only useful near perigee of the high eccentricity THEMIS orbit. Near perigee, ephemeris errors result in significant errors in the reference magnetic field. These reference field errors are the major contributors to the THEMIS SpinKF attitude error.

BIN-TRANSITION, SUN ONLY ATTITUDE ACCURACY USING SIMULATED DATA

With simulated data the accuracy of Fuzzycones Sun-only attitude determination could be evaluated over a wide range of spin-axis positions. The results are expected to be accurate as long as the noise model used is accurate.

Simulations were run for a number of spin-axis positions with angles from the ecliptic plane ranging from 0 to 90 degrees, and rotations about the ecliptic pole ranging from 0 to 90 degrees from the Sun position. For each attitude, BTS angles were estimated by applying Gaussian noise to the "true" Sun angle for two adjacent bin transitions. These were used to determine an attitude in the same manner as they are in the Fuzzycones algorithm. One thousand attitude solutions were determined for each "truth" attitude and the root-mean-square (RMS) attitude errors were determined. These RMS attitude errors are displayed in Table 2 and presented in Fig. 10.

The unevenness in the plot near the ecliptic plane, when the rotation about the pole is large, is a result of the merging of Gaussian distributions. When the separation between Gaussian distributions equals their uncertainty, their two maxima abruptly merge into a single maximum at their mean. The difference between the "truth" and computed attitudes abruptly increases. This effect causes the unevenness in the plot.

Table 2. Two-bin-transition Fuzzycones Attitude Errors for Several Spin-Axis Positions (Spin-Axis Positions Represented by Angles from Ecliptic Plane and Rotation About Ecliptic Pole from Spin-Axis to Spacecraft-Sun Direction)

Rotation About Ecliptic Pole (deg)	Angle from Ecliptic Plane (deg)								
	0	10	20	30	40	50	60	70	80
0	0.14	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
10	2.38	0.38	0.29	0.25	0.23	0.19	0.14	0.10	0.25
20	4.64	1.18	0.71	0.58	0.45	0.37	0.29	0.20	0.10
30	6.99	2.42	1.30	0.93	0.74	0.55	0.43	0.29	0.14
40	8.77	3.99	1.95	1.33	1.03	0.76	0.53	0.37	0.19
50	10.42	6.18	2.86	1.73	1.31	0.95	0.70	0.44	0.22
60	12.11	7.95	3.40	2.22	1.54	1.12	0.815	0.51	0.26
70	12.73	9.27	4.09	2.51	1.75	1.17	0.88	0.55	0.27
80	13.23	10.25	4.45	2.79	1.84	1.30	0.94	0.58	0.29
90	13.63	10.68	4.52	2.86	1.94	1.37	0.93	0.60	0.29

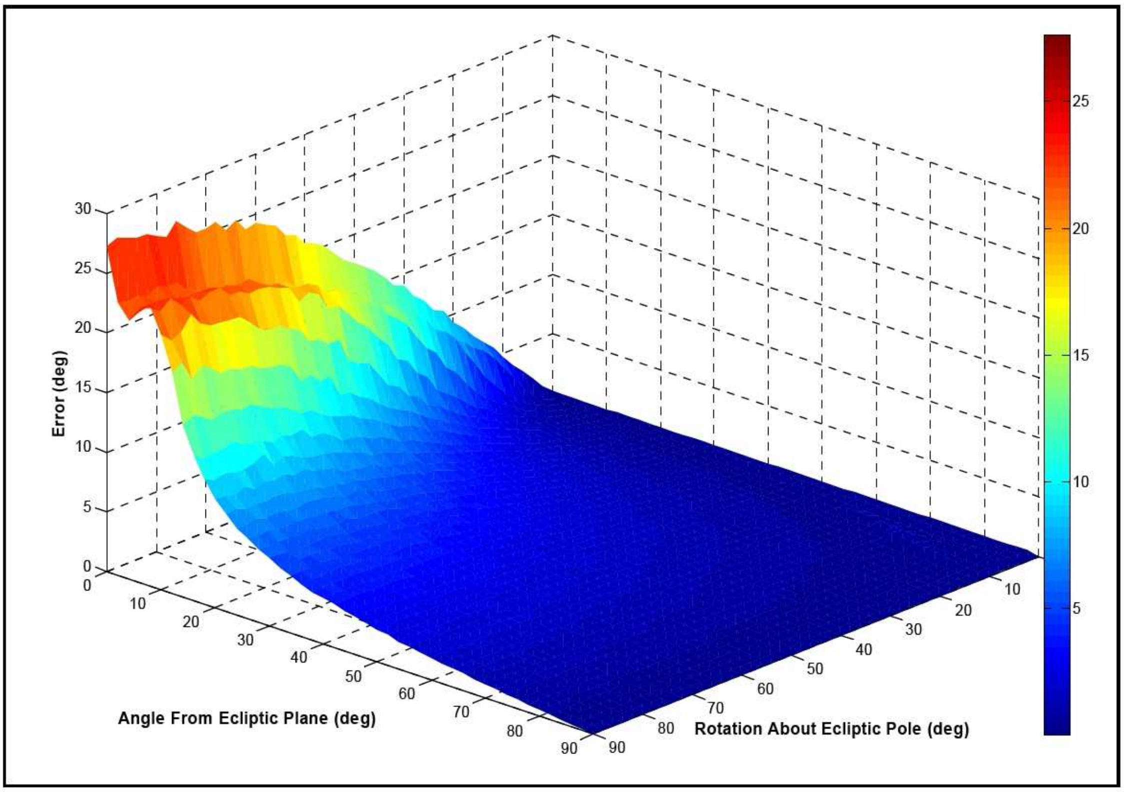

Fig. 10. Two-bin-transition Fuzzycones Attitude Errors as a Function of Spin-Axis Positions (Spin-Axis Positions Represented by Angles from Ecliptic Plane and Rotation About Ecliptic Pole from Spin-Axis to Spacecraft-Sun Direction

TIME BETWEEN BIN TRANSITIONS

In order to obtain sufficient attitude accuracy to perform the maneuvers needed to move from the THEMIS to the ARTEMIS orbits, two bin transitions must occur in the time between maneuvers (a few days). Fortunately a simple relation exists between the Sun angle with time and the spin-axis direction.

$$\rho = \pi/2 + \sin(\Delta\Phi)(\pi/2 - \delta) \tag{10}$$

For a constant spin axis direction (specified by [δ,Φ]), the term $\Delta\Phi$ varies with time since the spacecraft to Sun vector's right ascension varies with time. Using this, the number of days between bin transitions can be estimated. Such an estimate, using the THEMIS geometry, is shown in Fig. 11.

As can be seen in this figure, the interval needed to obtain two bin transitions is less than two days for most of the year. There are two periods, each of about two weeks duration, during which longer intervals are needed. However, since the position of these periods depends on the difference in right ascension of the spin-axis and the Sun, rotating the spin axis about the ecliptic pole by less than 15 degrees reduces the interval to less than 2 days.

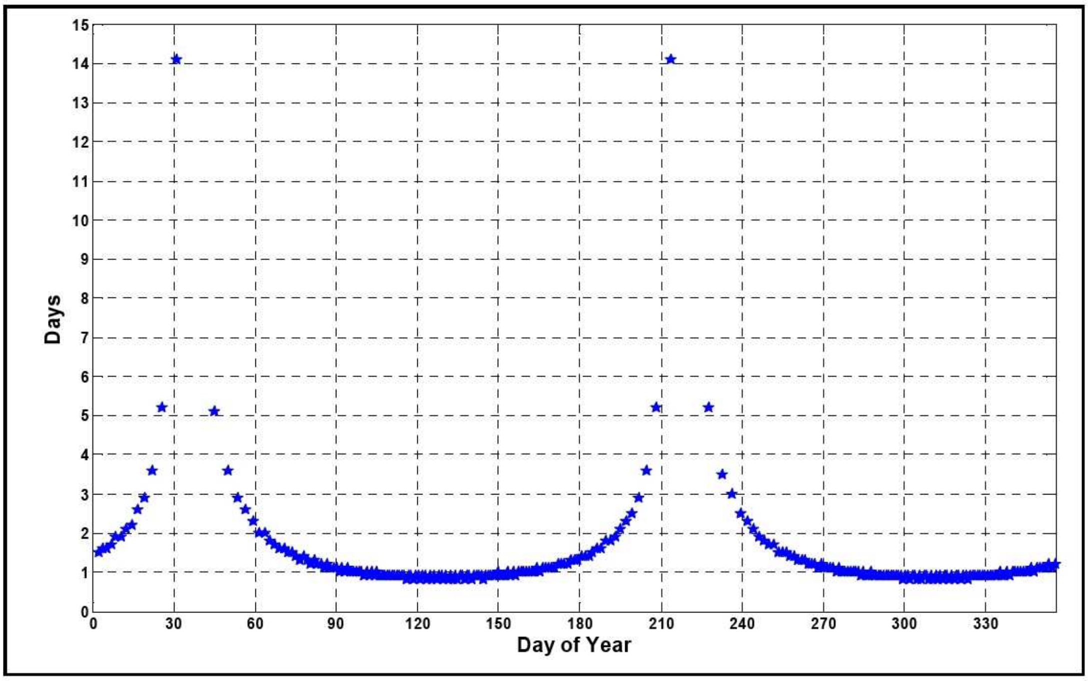

Fig. 11. Minimum Days Needed to Observe Two Bin Transitions for Artemis Geometry

CONCLUSIONS

A number of significant conclusions can be drawn from this analysis. Perhaps the most important for the ARTEMIS mission is that it is practical to obtain attitudes using only the Sun sensor with an accuracy sufficient for mission success. The minimum required accuracy has not been well defined but a value of about 1 degree is accepted as safe. Orbit maneuvers are planned at nominal intervals of no less than 2 orbit periods (but never less than 1 orbit period). The initial orbital periods of the two THEMIS probes to be converted to ARTEMIS are two and four days, and as orbit raising maneuvers are performed, the periods will increase. Thus, the times between which attitudes of 1 degree or better must be obtained are nominally more than four days and never less than two days. Using two or more bin transitions, Fuzzycones will easily meet this requirement.

Fuzzycones is a useful technique for determination of the attitudes of spin-stabilized spacecraft and it is amenable to use with relatively short spans of Sun-only data. It is less useful, however, if the major measurement uncertainty has a uniform distribution (digitization error). In cases with only uniform distribution data, the cones method provides attitudes that are as accurate as those from Fuzzycones.

Use of bin-transition data significantly increases the accuracy of Sun-only attitude solutions with errors arising primarily from digitization. In these cases, Fuzzycones may be used with the bin-transition data alone or combined with the measured Sun angles to provide accurate attitudes. The accuracy of such Fuzzycones solutions depends on the direction of the spin axis relative to the Sun. Errors are largest when the spin-axis is near the ecliptic and pointing near the Sun, and smallest when the spin axis is near the ecliptic pole.

REFERENCES

[1] J.A. Hashmall, *Accommodating Sensor Uncertainty in the Cones Method: Polycones and Fuzzycones*, AIAA 2006-6162 ,AIAA/AAS Astrodynamics Specialist Conference, Keystone, CO, August 2006

[2] G. Natanson, J.E. Sedlak, and J.A. Hashmall, *Mission Experience Using New Attitude Tools for Spinning Spacecraft*, AIAA-2008-6600, AIAA/AAS Astrodynamics Specialist Conference and Exhibit, Honolulu, HI, August, 2008

[3] V. Angelopoulos and D. G. Sibeck*, Time History of Events and Macroscale Interactions during Substorms: Themis, and Acceleration, Reconnection Turbulence, and Electrodynamics of Moon's Interaction with the Sun: Artemis, Proposal submitted for: Senior Review 2008of the mission operations and data analysis program for the Heliophysics operating missions* pg, 1, URL: http://www.igpp.ucla.edu/public/THEMIS/SCI/Pubs/Proposals%20and%20Reports/HP_SR_2008_THEMIS_SciTech_20080221.pdf

[4] F.L. Markley and J.E. Sedlak, *Kalman Filter for Spinning Spacecraft Attitude Estimation*, Journal Of Guidance, Control, And Dynamics, pp 1750-1760, Vol.31, No.6, Nov.-Dec. 2008

[5] J.R. Wertz (ed.), *Spacecraft Attitude Determination and Control*, Chap. 11, D. Reidel, Dordrecht, Holland, 1978

[6] D. Mortari and P. Singla, *Optimal Cones Intersection Technique*, pp. 474 – 482,Acta Astronautica, Vol. 59, 2006

[7] J.C. van der Ha, *Two-Sun-Cones Attitude-Determination Method*, Journal Of Guidance, Control, And Dynamics, pp 1202-1209, Vol.31, No.5, Sep.-Oct. 2008

ACKNOWLEDGEMENTS

The authors wish to thank Daniel Cosgrove of the University of California (Berkeley) Space Sciences Laboratory for providing the THEMIS data used in this work.

The authors acknowledge support from the National Aeronautics and Space Administration (NASA) Mission Operations and Missions Services (MOMS) Contract NNG04DA01C, Task Order 088.